

Chapter 8 — Multi-Step Percent Problems — Teacher Guide

Chapter at a glance

Lesson duration	4 lessons, 45–65 minutes each (see pacing table below)
Materials	Original-change-final strips, base cards, price cards, percent cards, calculators for verification, year-by-year interest tables, the two chapter diagrams
Launch question	“A \$100 item is 20% off, then another 20% off. Is the final price \$60, or something else?”
Key representation	The base-tracking strip — keep one visible throughout instruction
Anticipated responses	Correct: names the new base after each step before applying the next percent. Partial: calculates one step correctly but reuses the original base for the second. Incorrect: adds the percentages together (for example, “20% + 20% = 40% off”).
If the readiness check fails	Reteach one-step percent change from Chapter 7 before starting the Launch; do not skip the base-tracking table
Talk prompt	“What amount is 100% at this step?”
Exit ticket	5 items — discount then tax, explain why successive discounts don’t add, reverse percent, simple interest, state the base-tracking routine (see Section 14)
Reteach / extend	Reteach: single successive-discount problems only, with a two-row base table. Extend: composed multipliers, reverse engineering, order-effect comparisons.

Purpose and pacing

Chapter 8 is the book’s first sustained treatment of composition: a percent change creates a new base for the next action. Students should physically or pictorially record each new amount before moving on.

Lesson	Focus	Suggested time
Lesson 1	Readiness, launch, successive discounts, and base tracking	45–60 minutes
Lesson 2	Discount then tax, increase then decrease,	45–60 minutes

Lesson	Focus	Suggested time
	and multiplier products	
Lesson 3	Commission, simple interest, reverse percent, and error diagnosis	45–60 minutes
Lesson 4	Mixed practice, Tripwire, Exit Ticket, and targeted reteaching	50–65 minutes

Materials

Prepare original-change-final strips, base cards, price cards, percent cards, calculators for verification, year-by-year interest tables, and the chapter diagrams. Make students physically move the “base” card after each step.

Teacher launch guidance

Use the \$100 item with two 20% discounts. Ask students to solve mentally and record their predicted final price. Then compare the three claims. The purpose is not merely to teach a correction; it is to expose why the second percent is applied to the new base.

For discount-then-tax, state the tax base explicitly at first. Later, present problems where the wording requires students to decide whether tax applies to the original price, sale price, or another stated amount. The phrase “on the sale price” should be treated as mathematically important.

Teacher prompts

Purpose	Prompt
Name the base	“What amount is 100% at this step?”
Sequence steps	“What is the new amount before the next percent?”
Compare methods	“What multiplier represents this change?”
Diagnose a shortcut error	“Did the second percent act on the original or new base?”
Support reverse percent	“What final percent remains after the discount?”
Support interest	“Is the rate applied once, each year to the principal, or to a growing balance?”
Check result	“Does the direction and size of each change match the final amount?”

Differentiation

Students needing concrete support

Use \$100 examples and physical strips. Have students record each step in a table with columns for base, percent, change, and new base. Use one operation per line and delay multiplier notation until the sequence is secure.

Students needing language support

Use the frames:

- “At this step, the base is _____.”
- “The change is _____% of _____.”
- “The new base is _____.”
- “I cannot add the percentages directly because _____.”
- “The final answer is reasonable because _____.”

Students ready to extend

Introduce composed multipliers, reverse engineering, simple versus compound growth, and order effects. Ask students to compare the outcomes of two changes in different orders and explain why multiplication of the factors is commutative while the contextual base interpretation still matters.

Formative assessment

Record whether students name the base, calculate the percent amount, update the base, preserve units, and check the sequence. A correct final answer without a base-tracking record should be considered partial evidence on a reasoning task.

Revision Support: Expected Ideas and Teacher Moves

Use this compact guide with the new Cumulative Connection and Strategy Studio tasks. Accept equivalent representations when the relationship, units, and reasoning are correct.

Cumulative Connection: Identify the base after a first discount. Write a multiplier for a 15% increase. Estimate the result of 80 reduced by 25% and then increased by 10%. Explain why the original amount may not be the base at every step.

Strategy Studio guidance: Expected ideas: $\$120 \times 0.75 \times 1.08 = \97.20 ; reverse discount requires $64 \div 0.80 = \$80$; equivalent multipliers can come from different sequences; each step must name its current base. Encourage a timeline or factor chain.

Transfer Bridge Teacher Guidance

Use the four tasks to distinguish imitation from transfer. Ask students to name the relationship, keep units visible, compare representations, and explain or repair an error. Accept equivalent methods when the structure and interpretation are correct.

Expected guidance: Familiar: $\$120 \times 0.75 \times 1.08 = \97.20 . Altered: $\$64 \div 0.80 = \80 . Non-routine: $0.80 \times 0.90 = 0.72$, so the two discounts equal a 28% reduction, not 30%. Repair: each percentage uses the current base unless the problem explicitly states another base.

Record whether the difficulty is classification, representation, operation, computation, units, interpretation, or checking. Do not assign more routine practice until the source of the error is identified.

Discussion and evidence note

Ask students to name the relationship before calculating, compare at least two representations when appropriate, and revise an explanation after hearing another

strategy. Record whether the difficulty is classification, representation, operation, computation, units, or checking.

Answer Key

Ready-to-Learn Check

1. 20.
2. \$80.
3. 8.
4. \$66.
5. 30.
6. The new amount after the first change.
7. Principal, rate, and time.

Worksheet A

1. **\$45.** Likely wrong answer: \$50 (reports the starting amount, not recognizing that the discount must be subtracted before the tax step). Misconception: not updating the base after the first change. Next move: "what is \$50 minus 10% of \$50?"
2. **\$80 is taxed.** Likely wrong answer: \$100 (calculates tax on the original price instead of the sale price). Misconception: forgetting the new base — the chapter's central error. Next move: "what amount remains after the 20% discount? That's what gets taxed."
3. **\$220.** Likely wrong answer: 200 (uses the original value instead of the increased value as the base for the decrease). Misconception: not tracking the base across steps. Next move: "what is the value after the 10% increase? That's the base for the decrease."
4. **\$600 each year.** Likely wrong answer: a growing balance (for

example, \$630 in year 2), applying compound rather than simple interest. Misconception: confusing simple interest (same principal every year) with compound interest (growing balance). Next move: "does simple interest use the original principal, or last year's balance?"

5. **\$800 sales.** Likely wrong answer: \$56 (reports the commission amount instead of identifying the sales total as the base). Misconception: confusing the commission with the base it's calculated from. Next move: "what amount is 7% being taken from?"
6. **90% of the original.** Likely wrong answer: 80% (uses the discount rate itself as if it were the remaining percent, instead of subtracting from 100%). Misconception: confusing the percent removed with the percent remaining. Next move: "if 10% is taken off, what percent is left?"
7. **\$49.50.** Likely wrong answer: \$45.50 (adds a flat \$4.50 instead of computing 10% of the new \$45 base). Misconception: not recalculating the percent amount from the updated base. Next move: "what is 10% of \$45, exactly?"
8. **\$84.** Likely wrong answer: \$85 (computes 5% of the original \$100 instead of 5% of the \$80 sale price). Misconception: taxing the wrong base. Next move: "5% of which amount — \$100 or \$80?"
9. **\$198.** Likely wrong answer: 200 (assumes the 10% increase and 10% decrease cancel out). Misconception: assuming equal percent changes in opposite directions return to the start. Next

- move: “is 10% of 220 the same as 10% of 200?”
10. **\$60 interest.** Likely wrong answer: \$30 (uses only one year’s worth of interest, forgetting to multiply by the number of years). Misconception: forgetting time in the interest formula. Next move: “the rate is per year — how many years are there?”
 11. **\$56 commission.** Likely wrong answer: \$800 (reports the sales total as if it were the commission itself). Misconception: confusing the base with the amount calculated from it. Next move: “what is 7% of \$800, not \$800 itself?”
 12. **\$80.** Likely wrong answer: \$79.20 (multiplies \$72 by 0.10 and subtracts, treating \$72 as if it still needed a discount applied, instead of dividing to reverse one). Misconception: applying the forward operation instead of reversing it. Next move: “if \$72 is already the discounted price, what operation undoes a discount?”
 13. **The first change creates a new amount, which becomes the base for the next step.** Common flaw: states that “the base changes” without explaining why or what it becomes. Next move: ask for the specific new base value in a given example.
 14. **The problem defines the sale price as the tax base; therefore tax is calculated on the new sale-price amount.** Common flaw: says tax “just goes on the new price” without connecting it to what the problem’s wording specifies. Next move: ask the student to quote the exact phrase in the problem that names the tax base.
1. **Discount: \$30; sale price \$90; tax \$10.80; final \$100.80.** Likely wrong answer: tax calculated on \$120 instead of \$90, giving a different final total. Misconception: taxing the original price instead of the sale price. Next move: “what amount does the 12% tax apply to?”
 2. **Sale price \$63; tax \$6.30; final \$69.30.** Likely wrong answer: tax calculated on \$90 instead of \$63. Misconception: same as #1. Next move: same check.
 3. **$400 \times 1.20 \times 0.75 = 360$.** Likely wrong answer: $400 \times 0.95 = 380$ (adds the 20% and subtracts the 25% as if they combined into a single 5% net change). Misconception: treating successive percent changes as additive rather than multiplicative. Next move: “does the second change act on 400, or on the already-changed amount?”
 4. **$500 \times 0.90 \times 1.10 = 495$.** Likely wrong answer: 500 (assumes the 10% decrease and 10% increase cancel out exactly). Misconception: assuming equal-percent opposite changes return to the start. Next move: “is 10% of 450 the same amount as 10% of 500?”
 5. **$\$700 \times 0.06 \times 4 = \168 interest.** Likely wrong answer: \$42 (calculates only one year of interest, forgetting to multiply by 4). Misconception: forgetting time in the simple interest formula. Next move: “the rate is per year — how many years?”
 6. **\$96 commission.** Likely wrong answer: \$1,200 (reports the sales total instead of 8% of it). Misconception: confusing the base with the commission calculated from it. Next move: “what is 8% of \$1,200, not \$1,200 itself?”

Worksheet B

7. **$85 \div 0.85 = 100$.** Likely wrong answer: $85 \times 0.85 = 72.25$ (multiplies instead of dividing to reverse the percent). Misconception: applying the forward operation instead of reversing it. Next move: "if 85 is already 85% of the original, what operation finds the original?"
 8. **$56 \div 0.70 = 80$.** Likely wrong answer: $56 \times 0.70 = 39.20$ (same reversal error as #7). Misconception: same as #7. Next move: same check.
 9. **$120 \times 0.75 \times 1.12 = 100.80$.** Common flaw: computes the two steps separately (as in #1) but makes an arithmetic slip combining them into a single multiplier. Next move: check that $0.75 \times 1.12 = 0.84$ before multiplying by 120.
 10. **The second 10% increase applies to 450, not 500; the final is 495.** Common flaw: states the final answer is different from 500 without explaining which base the second step actually used. Next move: ask for the intermediate value after the first step.
 11. **Answers should show $400 \rightarrow 480 \rightarrow 360$.** Common flaw: the table skips the intermediate value (480) and jumps straight to the final answer. Next move: require every intermediate base to be recorded.
 12. **$0.70 \times 80 = 56$.** Common flaw: checks by repeating the original division instead of using the inverse operation (multiplication) to confirm. Next move: ask for a check that starts from the answer, not a repeat of the original steps.
- $\$300$ instead of the marked-up $\$360$.** Misconception: not updating the base after the markup step. Next move: "what amount is 15% of — the original cost, or the marked-up price?"
 2. **$1,000 \times 1.12 \times 0.95 = 1,064$.** Likely wrong answer: $1,000 \times 1.07 = 1,070$ (adds 12% and subtracts 5% as if they combined additively). Misconception: treating successive percent changes as additive. Next move: "does the 5% decrease act on 1,000, or on the grown population?"
 3. **$96 \div 0.80 = \$120$.** Likely wrong answer: $96 \times 0.80 = 76.80$ (multiplies instead of dividing to reverse the discount). Misconception: applying the forward operation instead of reversing it. Next move: "if \$96 is already 80% of the original, what operation finds the original?"
 4. **$\$100 \rightarrow \$110 \rightarrow \$99$, so it does not return to the original.** Likely wrong answer: agrees with the student's claim that it returns to \$100. Misconception: assuming equal-percent opposite changes cancel out. Next move: "is 10% of 110 the same amount as 10% of 100?"
 5. **$100 \times 0.80 \times 1.10 = \88 ; combined multiplier 0.88.** Likely wrong answer: tax calculated on the original \$100 instead of the \$80 sale price. Misconception: not tracking the base across steps. Next move: "what amount does the 10% tax apply to?"
 6. **$\$120$.** Likely wrong answer: \$2,400 (reports the sales total instead of 5% of it). Misconception: confusing the base with the commission calculated from it. Next move: "what is 5% of \$2,400, not \$2,400 itself?"

Worksheet C

1. **\$360 after markup; \$54 discount; final \$306.** Likely wrong answer: discount calculated on the original

7. **Interest $\$900 \times 0.03 \times 5 = \135 ; final $\$1,035$.** *Likely wrong answer: $\$27$ (calculates one year only, forgetting to multiply by 5). Misconception: forgetting time in the interest formula. Next move: “the rate is per year — how many years?”*
8. **The second 10% applies to the already increased amount; combined multiplier is $1.25 \times 1.10 = 1.375$, or a 37.5% increase.** *Likely wrong answer: “35%,” adding the two percents directly. Misconception: treating successive percent increases as additive. Next move: “does the second 10% act on the original amount, or the amount after the first increase?”*
9. **Answers vary but must track the new base.** *Common flaw: creates a discount-then-tax problem but applies the tax to the original price instead of the sale price. Next move: check that the second step’s base is explicitly named.*
10. **Answers vary but must use the final percent to reverse the last change.** *Common flaw: creates a reverse-percent problem but solves it by multiplying instead of dividing. Next move: check that the solution divides by the remaining percent (as a decimal).*
11. **$\$145.80$. The length of the sale (2 days) and the number of other discounted items (20) were not needed to find the final price.** *Likely wrong answer: works the 2 days or 20 items into the calculation somehow. Misconception: assuming every number in a word problem must be used. Next move: “what does the question actually ask for? Which numbers answer that?”*
12. **Answers vary; must describe a situation with two successive 20% decreases (for example, two rounds of depreciation or two successive discounts), with each 0.80 representing “80% remains after a 20% decrease.”** *Common flaw: describes only one step instead of two, or confuses 0.80 with a 20% amount rather than the multiplier for “80% remains.” Next move: ask the student to state what each individual 0.80 means before combining them.*

Extend Your Thinking

1. The second 20% is taken from the reduced amount, so the final is 64% of the original, not 60%.
2. Yes, $1.25 \times 0.80 = 1$.
3. Starting at 100: increase then decrease gives $100 \times 1.20 \times 0.80 = 96$; decrease then increase gives $100 \times 0.80 \times 1.20 = 96$. Both orders give the same result, 96, because multiplication can be done in either order — but neither returns to the original 100.
4. $\text{Original} \times 1.10 \times 1.10 \times 1.10$.
5. Simple interest uses the original principal each period; repeated-percent growth applies each percent to the new balance.
6. Use $72 \div 0.80 = 90$, or reason that 80% is 72 and 10% is 9, so 100% is 90.

Tripwire and Exit Ticket

Tripwire: $\$202.50$.

Exit Ticket: $\$88$;

successive discounts use a new base, so the final is 64% rather than 60%;

original price $\$90$;

interest $\$60$;

“After each change, name the new amount and use it as the next base unless the context states otherwise.”

